

Exercise 13

Differentiate.

$$y = \frac{t \sin t}{1 + t}$$

SolutionUse the product and quotient rules to differentiate y .

$$\begin{aligned} y' &= \frac{dy}{dt} \\ &= \frac{d}{dt} \left(\frac{t \sin t}{1 + t} \right) \\ &= \frac{\left[\frac{d}{dt}(t \sin t) \right] (1 + t) - \left[\frac{d}{dt}(1 + t) \right] (t \sin t)}{(1 + t)^2} \\ &= \frac{\left\{ \left[\frac{d}{dt}(t) \right] \sin t + t \left[\frac{d}{dt}(\sin t) \right] \right\} (1 + t) - (1)(t \sin t)}{(1 + t)^2} \\ &= \frac{[(1) \sin t + t(\cos t)] (1 + t) - t \sin t}{(1 + t)^2} \\ &= \frac{(\sin t + t \cos t + t^2 \cos t + t \sin t) - t \sin t}{(1 + t)^2} \\ &= \frac{\sin t + t \cos t + t^2 \cos t}{(1 + t)^2} \end{aligned}$$